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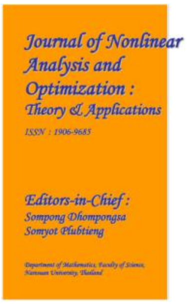

PERISHABLE PRODUCTION INVENTORY SYSTEM WITH MACHINE BREAK DOWNS AND NEGATIVE CUSTOMERS

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ABSTRACT
This study examines a production inventory system with machine break downs including perishable items and negative customers. Customers are considered ordinary when they arrive and increase their level in orbit with a probability of p. Conversely, a customer who drops their level without inventory with a probability of 1-p is referred to as a negative customer. The completion of service makes one customer leave the system and decreases the inventory level by one. When any item from the level of inventory that is on hand perishes. The maximum inventory level is set as S. (s,S) policy is the production policy. It is possible for a machine to break down during production; it will eventually be fixed at random. In the steady-state sce nario, the joint probability distribution of the number of customers in the orbit, inventory level, and machine condition is obtained. Next, we calculate a measure of performance and use them as the foundation for creating a cost function. Lastly, a few numerical findings are shown.
Keywords: Production Inventory System, Service facility, Negative Customers, (s,S) Policy, Perishable items, Machine Breakdown.

1. INTRODUCTION
Buzacott and Shanthikumar [4] provide an in-depth analysis of stochastic models in manufacturing systems. Groenevelt et al. [2] discussed about a production batching with machine breakdowns and safety stocks. He et al. at [5] considered a production inventory system with arrival of demands forming a Poisson process, production time having exponential distribution and lead time is zero, so that the production can be started at any required time. Perumal and Arivarignan discuss a Production Inventory Model with Two rates of Productions and backorders [7]. Berman and Sapna [6] discussed the optimal service rates for a facility that stocks goods that are perishable. Manuel et al. [10] speak about a perishable inventory system with service facilities and retrial customers. Anita et al. [11] worked on the production inventory system with machine breakdown; in this study, they used an exponential distribution for production time, inter-failure time, and repair time. The production inventory system was described by Karthick et al. [12] along with two different customer types and machine breakdowns. In [13], KY Kung et al. talked about the production inventory system for degrading items that includes inspection, partial backordering, and machine breakdowns. Sunjida Haque et al. [14] investigate the perishable stochastic inventory System in an infinite pool system. Gelenbe [1] introduced the concept of negative customer in the queueing network. The dependence between positive arrival and negative arrival of customers has been introduced by Shin and Choi [8]. Sivakumar and Arivarignan [9] gives a detailed account on a perishable inventory system at service facilities with negative customers. In this paper we combine a perishable production inventory system with machine break downs and negative customer.

2. MODEL DESCRIPTION
We examine a production inventory system with a maximum capacity of S units in stock. The probability of an arriving customer increasing their level in the orbit is p, whereas the chance of a customer decreasing by one without inventory (negative customer) is $1 - p = q$. Assumed to be

$p\lambda$ for the arrival rate and $q\lambda$ for the negative rate of customers. When a customer lowers by one with a unit item, the service rate occurs. With a service rate of δ, the service facility uses exponential distribution. When any item from the level of inventory that is on hand perishes, the perishable rate occurs. This rate is determined by an exponential distribution with a parameter of θ. Once the inventory falls below a certain threshold, such as $s(< S)$ the machine is activated to make the item. Assumed to be exponentially distributed with parameter ς is the production of a unit item.When the inventory level reaches S, manufacturing is halted. The machine may break down during production, in which case an exponential distribution with parameter ϑ may be assumed. The breakdown machine is repaired after a random amount of time, and the duration of the repair is determined by the exponential distribution with parameter ϱ.

3. MATHEMATICAL ANALYSIS

Let U(t) be number of customers in the orbit at time t, V(t) be on-hand inventory level at time t and W(t) be the status of the machine at time t.

$$W(t) = \begin{cases} 0, & \text{the machine is idle} \\ 1, & \text{the machine is switched on} \\ 2, & \text{the machine is under repair.} \end{cases}$$

It can be demonstrated that the stochastic process is a continuous time Markov chain with state space $\{(U(t),V(t),W(t)),t\geq 0\}$ based on the assumptions made about the input and output processes.

$$\Omega = \begin{cases} (u,v,0), & u = 0,1,2,\ldots; \quad v = s+1, s+2, \ldots, S; \\ (u,v,w), & u = 0,1,2,\ldots; \quad v = 0,1,2,\ldots,S-1; \quad w = 1,2,; \end{cases}$$

The ordering of the above states space is denoted by, ($\ll 0 \gg$, $\ll 1 \gg$, $\ll 2 \gg$, ...), where

$$\ll u \gg \; = \; (< u,0 >, < u,1 >, \ldots, < u,S >), \quad u = 0,1,2,\ldots$$

and

$$< u,v > \; = \begin{cases} (< u,v,1 >, < u,v,2 >), \quad v = 0,1,2,\ldots,s; \\ (< u,v,0 >, < u,v,1 >, < u,v,2 >), v = s+1, s+2, \ldots, S-1; \\ < u,v,0 >, \quad v = S; \end{cases}$$

Due to the possibility of a single customer increase, decrease or stay constant with in the orbit, the infinitesimal generator matrix T (block matrix) is a tridiagonal matrix.

Let

$$T = \begin{array}{c c} & \begin{array}{cccccc} \ll 0 \gg & \ll 1 \gg & \ll 2 \gg & \ll 3 \gg & \ll 4 \gg & \cdots \end{array} \\ \begin{array}{c} \ll 0 \gg \\ \ll 1 \gg \\ \ll 2 \gg \\ \ll 3 \gg \\ \vdots \end{array} & \begin{pmatrix} B_0 & A_0 & 0 & 0 & 0 & \cdots \\ A_2 & A_1 & A_0 & 0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{pmatrix} \end{array}$$

where

$$[A_0]_{vv'} = \begin{cases} A_{00}, & v' = v, \quad v = 0,1,2,\ldots,s; \\ A_{01}, & v' = v, \quad v = s+1,\ldots,S-1; \\ A_{02}, & v' = v, \quad v = S; \\ \mathbf{0} & \text{otherwise.} \end{cases} , \quad A_{00} = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} p\lambda & 0 \\ 0 & p\lambda \end{pmatrix} \end{array} A_{01} = \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} p\lambda & 0 & 0 \\ 0 & p\lambda & 0 \\ 0 & 0 & p\lambda \end{pmatrix} \end{array}, A_{02} = \begin{array}{cc} & 0 \\ 0 & (p\lambda) \end{array},$$

$$[A_2]_{vv'} = \begin{cases} \tilde{A}_{20}, & v' = v-1, \quad v = 1,2,\ldots,s; \\ \tilde{A}_{21}, & v' = v-1, \quad v = s+1; \\ \tilde{A}_{22}, & v' = v-1, \quad v = s+2, s+3, \ldots, S-1; \\ \tilde{A}_{23}, & v' = v-1, \quad v = S; \\ A_{20}, & v' = v, \quad v = 0,1,2,\ldots,s; \\ A_{21}, & v' = v, \quad v = s+1, s+2, \ldots, S-1; \\ A_{22}, & v' = v, \quad v = S; \\ \mathbf{0} & \text{otherwise.} \end{cases} \quad \tilde{A}_{20} = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix} \end{array}, \quad \tilde{A}_{21} = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} \delta & 0 \\ \delta & 0 \\ 0 & \delta \end{pmatrix} \end{array},$$

$$\tilde{A}_{22} = \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} \delta & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \delta \end{pmatrix} \end{array}, \tilde{A}_{23} = \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ 0 & \begin{pmatrix} \delta & 0 & 0 \end{pmatrix} \end{array}, A_{20} = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} q\lambda & 0 \\ 0 & q\lambda \end{pmatrix} \end{array}, A_{21} = \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} q\lambda & 0 & 0 \\ 0 & q\lambda & 0 \\ 0 & 0 & q\lambda \end{pmatrix} \end{array}, A_{22} = \begin{array}{cc} & 0 \\ 0 & (q\lambda) \end{array},$$

$$[B_0]_{vv'} = \begin{cases} \tilde{B}_{01}, & v' = v-1, \quad v = 1,2,\ldots,s; \\ \tilde{B}_{02}, & v' = v-1, \quad v = s+1; \\ \tilde{B}_{03}, & v' = v-1, \quad v = s+2,\ldots,S-1; \\ \tilde{B}_{04}, & v' = v-1, \quad v = S; \\ B_{00}, & v' = v, \quad v = 0; \\ B_{11}, & v' = v, \quad v = 1,2,\ldots,s; \\ B_{22}, & v' = v, \quad v = s+1,s+2,\ldots,S-1; \\ B_{33}, & v' = v, \quad v = S; \\ B_{01}, & v' = v+1, \quad v = 0,1,2,\ldots,s-1; \\ B_{02}, & v' = v+1, \quad v = s; \\ B_{03}, & v' = v+1, \quad v = s+1,s+2,\ldots,S-2; \\ B_{04}, & v' = v+1, \quad v = S-1; \\ \mathbf{0} & \text{otherwise.} \end{cases} \quad \tilde{B}_{01} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} v\theta & 0 \\ 0 & v\theta \end{pmatrix} \end{array},$$

$$\tilde{B}_{02} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} v\theta & 0 \\ v\theta & 0 \\ 0 & v\theta \end{pmatrix} \end{array}, \quad \tilde{B}_{03} = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} v\theta & 0 & 0 \\ 0 & v\theta & 0 \\ 0 & 0 & v\theta \end{pmatrix} \end{array}, \tilde{B}_{04} = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ 0 & \begin{pmatrix} v\theta & 0 & 0 \end{pmatrix} \end{array}, \quad B_{00} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} -(p\lambda + \varsigma + \vartheta) & \vartheta \\ \varrho & -(p\lambda + \varrho) \end{pmatrix} \end{array},$$

$$B_{11} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} -(p\lambda + \varsigma + \vartheta + v\theta) & \vartheta \\ \varrho & -(p\lambda + \varrho + v\theta) \end{pmatrix} \end{array}, B_{22} = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} -(p\lambda + v\theta) & 0 & 0 \\ 0 & -(p\lambda + \varsigma + \vartheta + v\theta) & \vartheta \\ 0 & \varrho & -(p\lambda + \varrho + v\theta) \end{pmatrix} \end{array},$$

$$B_{33} = \begin{array}{c c} & 0 \\ 0 & \begin{pmatrix} -(p\lambda + v\theta) \end{pmatrix} \end{array}, B_{01} = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} \varsigma & 0 \\ 0 & 0 \end{pmatrix} \end{array}, \quad B_{02} = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 0 & \varsigma & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}, \quad B_{03} = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varsigma & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}, \quad B_{04} = \begin{array}{c c} & 0 \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 \\ \varsigma \\ 0 \end{pmatrix} \end{array},$$

$$[A_1]_{vv'} = \begin{cases} \tilde{B}_{01}, & v' = v-1, \quad v = 1,2,\ldots,s; \\ \tilde{B}_{02}, & v' = v-1, \quad v = s+1; \\ \tilde{B}_{03}, & v' = v-1, \quad v = s+2,\ldots,S-1; \\ \tilde{B}_{04}, & v' = v-1, \quad v = S; \\ A_{10}, & v' = v, \quad v = 0; \\ A_{11}, & v' = v, \quad v = 1,2,\ldots,s; \\ A_{12}, & v' = v, \quad v = s+1,s+2,\ldots,S-1; \\ A_{13}, & v' = v, \quad v = S; \\ B_{01}, & v' = v+1, \quad v = 0,1,2,\ldots,s-1; \\ B_{02}, & v' = v+1, \quad v = s; \\ B_{03}, & v' = v+1, \quad v = s+1,s+2,\ldots,S-2; \\ B_{04}, & v' = v+1, \quad v = S-1; \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

3.1 StabilityAnalysis

Now, let us examine the generating matrix A=A0+A1+A2 is provided by the

$$[A]_{vv'} = \begin{cases} D_0, & v' = v-1, \quad v = 1,2,\ldots,s; \\ D_1, & v' = v-1, \quad v = s+1; \\ D_2, & v' = v-1, \quad v = s+2,s+3,\ldots,S-1; \\ D_3, & v' = v-1, \quad v = S; \\ C_0, & v' = v, \quad v = 0; \\ C_1, & v' = v, \quad v = 1,2,\ldots,s; \\ C_2, & v' = v, \quad v = s+1,s+2,\ldots,S-1; \\ C_3, & v' = v, \quad v = S; \\ B_{01}, & v' = v+1, \quad v = 0,1,2,\ldots,s-1; \\ B_{02}, & v' = v+1, \quad v = s; \\ B_{03}, & v' = v+1, \quad v = s+1,s+2,\ldots,S-2; \\ B_{04}, & v' = v+1, \quad v = S-1; \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

where $D_i = \widetilde{A}_{2i} + \widetilde{B}_{0(i+1)}, i = 0,1,2,3$; $C_0 = A_{00} + A_{10} + A_{20}$; $C_1 = A_{00} + A_{11} + A_{20}$;
$C_2 = A_{00} + A_{12} + A_{21}$; $C_3 = A_{02} + A_{13} + A_{22}$;

The ergodic of the generating matrix A is evident from this structure. The probability distribution for a steady state exists.

Assume that Π represents the steady state probability distribution of A, fulfilling the equation.

$$\Pi A = 0 \quad \text{and} \tag{1}$$
$$\Pi \mathbf{e} = 1 \tag{2}$$

where the vector $\Pi = (\pi^{(0)}\ \pi^{(1)}\ \pi^{(2)} \ldots \pi^{(S)})$. From the above two equations, we get the following set of equations

$$\begin{aligned}
\pi^{(i)}C_0 + \pi^{(i+1)}D_0 &= \mathbf{0} & i &= 0,\\
\pi^{(i-1)}B_{01} + \pi^{(i)}C_1 + \pi^{(i+1)}D_0 &= \mathbf{0} & i &= 1, 2, \ldots, s-1,\\
\pi^{(i-1)}B_{01} + \pi^{(i)}C_1 + \pi^{(i+1)}D_1 &= \mathbf{0} & i &= s,\\
\pi^{(i-1)}B_{02} + \pi^{(i)}C_2 + \pi^{(i+1)}D_2 &= \mathbf{0} & i &= s+1,\\
\pi^{(i-1)}B_{03} + \pi^{(i)}C_2 + \pi^{(i+1)}D_2 &= \mathbf{0} & i &= s+2, s+3, \ldots, S-2,\\
\pi^{(i-1)}B_{03} + \pi^{(i)}C_2 + \pi^{(i+1)}D_3 &= \mathbf{0} & i &= S-1,\\
\pi^{(i-1)}B_{04} + \pi^{(i)}C_3 &= \mathbf{0} & i &= S.
\end{aligned}$$

Lemma 1. *The stability condition of the system under study is given by*

$$p\lambda \quad < \quad q\lambda + (1 - \pi^{<0>}\boldsymbol{e})\delta \tag{3}$$

Proof. From the well known results of Neuts [3] on the positive recurrence of A, we have

$$\Pi A_0 \mathbf{e} \quad < \quad \Pi A_2 \mathbf{e} \tag{4}$$

and by exploiting the structure of the matrices A_0, A_2 and Π stated result follows.

$$A_{10} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} -(\lambda + \varsigma + \vartheta) & \vartheta \\ \varrho & -(\lambda + \varrho) \end{pmatrix} \end{array}, \quad A_{11} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{pmatrix} -(\lambda + \delta + \varsigma + \vartheta + v\theta) & \vartheta \\ \varrho & -(\lambda + \delta + \varrho + v\theta) \end{pmatrix} \end{array},$$

$$A_{12} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} -(\lambda + \delta + v\theta) & 0 & 0 \\ 0 & -(\lambda + \delta + \varsigma + \vartheta + v\theta) & \vartheta \\ 0 & \varrho & -(\lambda + \delta + \varrho + v\theta) \end{pmatrix} \end{array},$$

$$A_{13} = \begin{array}{c} \\ 0 \end{array} \begin{array}{c} 0 \\ \begin{pmatrix} -(\lambda + \delta + v\theta) \end{pmatrix} \end{array},$$

3.2 Steady State Analysis

It is clear that the continuous-time Markov Process $\{(U(t), V(t), W(t)), t \geq 0\}$ with state space Ω is regular from the irreducible structure of the rate matrix T and the lemma 3.1. Hence the limiting distribution Φ

$$\Phi^{(u,v,w)} = \lim_{t \to \infty} \Pr[U(t) = u, V(t) = v, W(t) = w | U(0), V(0), W(0)]$$

exists and is independent of the initial state. That is $\Phi = (\Phi^{(0)}, \Phi^{(1)}, \ldots)$ satisfies

$$\Phi T = 0, \Phi \mathbf{e} = 1 \tag{5}$$

4. SYSTEM PERFORMANCE MEASURES

We derive numerous important system performance metrics in the steady state in this section. We create an extended total cost function using them.

4.1 Expected Inventory level

$$\aleph_I = \sum_{u=0}^{\infty} \sum_{v=1}^{S} v\, \Phi^{(u,v)} \boldsymbol{e}$$

4.2 Expected number of customers in the waiting hall

$$\aleph_O = \sum_{u=1}^{\infty} u \Phi^{(u)} \boldsymbol{e}$$

4.3 Expected value of arrivals of negative customers

$$\aleph_N = \sum_{u=1}^{\infty} q\lambda \Phi^{(u)}\boldsymbol{e}$$

4.4 Expected Production startup rate

$$\aleph_P = \sum_{u=0}^{\infty} (s+1)\theta\Phi^{(u,s+1,0)} + \sum_{u=1}^{\infty} \delta\Phi^{(u,s+1,0)}$$

4.5 Expected Repair rate

$$\aleph_R = \sum_{u=0}^{\infty}\sum_{v=0}^{S-1} \varrho\, \Phi^{(u,v,2)}$$

4.6 Expected Perishable rate

$$\aleph_{PR} = \sum_{u=0}^{\infty}\sum_{v=1}^{S} v\theta\, \Phi^{(u,v)}\boldsymbol{e}$$

5. COST ANALYSIS

The long-run total expected cost rate for this model is defined to be

$$\mathrm{TC}(\mathrm{s},\mathrm{S}) = c_h\aleph_I + c_s\aleph_P + c_o\aleph_O + c_n\aleph_N + c_r\aleph_R + c_{pr}\aleph_{PR}$$

where c_h : The inventory carrying cost per unit item per unit time

c_s : Production startup cost for per production initiation

c_o : Waiting cost of a customer in the orbit per unit time

c_n : Loss per unit time due to arrival of a negative customer

c_r : Machine service cost per repair per unit time

c_{pr} : Perishable cost per unit item per unit time

substituting the values of ℵ's we get the Total Expected Cost.

6. NUMERICAL ANALYSIS

We have not provided an analytical demonstration of the convexity of TC (s, S) for the model presented in this work. On the other hand, our extensive numerical case experience suggests that the function TC (s, S) is convex. Here, we obtain the optimal values of the total cost rate, s, and S (let's say TC*, s*, S*) by a simple numerical search process. A typical three-dimensional plot of the expected total cost function is given in figure 1. The optimal cost value TC* = 1.040022 is obtained at (s*, S*) = (4,13) for fixed $\lambda = 0.95$; $\delta = 1.5$; $p = 0.6$; $\varsigma = 1.03$; $\vartheta = 0.07$; $\varrho = 0.31$; $\theta = 0.006$; $c_h = 0.02$; $c_s = 1.5$; $c_o = 1.05$; $c_n = 0.017$; $c_r = 0.15$; $c_{pr} = 0.82$ We have examined the impact of changing cost rates and system param eters on optimal values. The following tables show some of our findings. The optimal cost rate is indicated by the bottom entry in each cell and the matching S* and s* are indicated by the higher entries. For fixed $c_h = 0.02$; $c_s = 1.5$; $c_o = 1.05$; $c_n = 0.017$; $c_r = 0.15$; $c_{pr} = 0.82$, we analyse tables 1, 2, and 3 and find that

- The optimal cost value increases when λ, β and θ increases
- The optimal cost value decreases when μ, α and γ increases
- S* monotonically increases when λ, θ and β increases
- S* monotonically decreases when γ increases
- s* monotonically increases when λ and β increases
- s* monotonically decreases when γ and θ increases.

From the table 4, 5 and 6 for fixed $\lambda = 0.95$; $\delta = 1.5$; $p = 0.6$; $\varsigma = 1.03$; $\vartheta = 0.07$; $\varrho = 0.31$; $\theta = 0.006$ we observe that

- The optimal cost value increases when c_h, c_s, c_o, c_n, c_r and c_{pr} increases
- S* monotonically decreases when c_h increases
- S* monotonically increases when c_s and c_o increases.

For every other parameter, the S* and s* are unaffected.

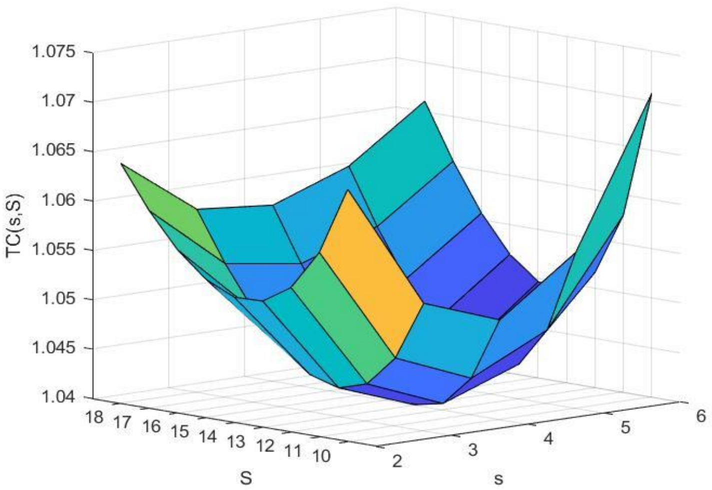

Figure 1: A typical three-dimensional plot for convexity of total expected cost rate

Table 1: Impact of Parameter on the Optimal Values for fixed λ=0.9

λ = 0.9	γ		0.308			0.31			0.312		
θ			0.0055	0.006	0.0065	0.0055	0.006	0.0065	0.0055	0.006	0.0065
μ	α	β									
1.45	1.02	0.06	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3
			1.0058	1.0071	1.0085	1.0051	1.0065	1.0078	1.0044	1.0058	1.0072
		0.07	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0139	1.0155	1.0170	1.0132	1.0148	1.0163	1.0125	1.0141	1.0156
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0220	1.0236	1.0252	1.0212	1.0228	1.0244	1.0204	1.0220	1.0236
	1.03	0.06	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3
			1.0047	1.0060	1.0074	1.0040	1.0054	1.0067	1.0034	1.0047	1.0061
		0.07	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0129	1.0145	1.0160	1.0123	1.0138	1.0153	1.0116	1.0131	1.0146
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0208	1.0224	1.0240	1.0201	1.0217	1.0232	1.0193	1.0209	1.0225
	1.04	0.06	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3
			1.0036	1.0050	1.0064	1.0030	1.0044	1.0057	1.0024	1.0037	1.0051
		0.07	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0120	1.0135	1.0150	1.0114	1.0129	1.0144	1.0107	1.0122	1.0137
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0197	1.0213	1.0229	1.0190	1.0206	1.0221	1.0183	1.0198	1.0214
1.5	1.02	0.06	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3
			0.9809	0.9823	0.9836	0.9802	0.9816	0.9829	0.9796	0.9809	0.9823
		0.07	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			0.9891	0.9906	0.9922	0.9884	0.9899	0.9914	0.9877	0.9892	0.9907
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9972	0.9989	1.0005	0.9964	0.9980	0.9996	0.9957	0.9972	0.9988
	1.03	0.06	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3
			0.9798	0.9811	0.9825	0.9791	0.9805	0.9818	0.9785	0.9798	0.9812
		0.07	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			0.9881	0.9896	0.9911	0.9874	0.9889	0.9904	0.9867	0.9882	0.9897
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9960	0.9976	0.9992	0.9953	0.9968	0.9984	0.9945	0.9961	0.9976
	1.04	0.06	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3	12 3
			0.9787	0.9801	0.9814	0.9781	0.9794	0.9808	0.9775	0.9788	0.9801
		0.07	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			0.9871	0.9886	0.9901	0.9865	0.9879	0.9894	0.9858	0.9873	0.9887

		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9949		0.9965		0.9980		0.9942		0.9957		0.9973		0.9934		0.9950		0.9965	
1.55	1.02	0.06	12	3	12	3	12	3	12	3	12	3	12	3	12	3	12	3	12	3
			0.9577		0.9591		0.9604		0.9570		0.9584		0.9597		0.9564		0.9577		0.9590	
		0.07	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			0.9660		0.9675		0.9690		0.9652		0.9667		0.9682		0.9645		0.9660		0.9675	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9742		0.9758		0.9773		0.9733		0.9749		0.9765		0.9725		0.9741		0.9757	
	1.03	0.06	12	3	12	3	12	3	12	3	12	3	12	3	12	3	12	3	12	3
			0.9566		0.9579		0.9592		0.9559		0.9572		0.9586		0.9552		0.9566		0.9579	
		0.07	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			0.9649		0.9664		0.9679		0.9642		0.9657		0.9672		0.9635		0.9650		0.9664	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9729		0.9745		0.9760		0.9721		0.9737		0.9752		0.9713		0.9729		0.9744	
	1.04	0.06	12	3	12	3	12	3	12	3	12	3	12	3	12	3	12	3	12	3
			0.9555		0.9568		0.9581		0.9548		0.9562		0.9575		0.9542		0.9555		0.9568	
		0.07	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			0.9639		0.9654		0.9669		0.9632		0.9647		0.9662		0.9625		0.9640		0.9655	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9718		0.9733		0.9748		0.9710		0.9725		0.9741		0.9702		0.9718		0.9733	

Table 2: Impact of Parameter on the Optimal Values for fixed λ=0.95

λ = 0.95		γ	0.308						0.31						0.312					
θ			0.0055		0.006		0.0065		0.0055		0.006		0.0065		0.0055		0.006		0.0065	
μ	α	β																		
1.45	1.02	0.06	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			1.0578		1.0593		1.0609		1.0571		1.0586		1.0602		1.0564		1.0579		1.0595	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0667		1.0683		1.0700		1.0659		1.0675		1.0692		1.0651		1.0667		1.0684	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0764		1.0782		1.0800		1.0755		1.0772		1.0790		1.0745		1.0763		1.0780	
	1.03	0.06	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			1.0568		1.0583		1.0598		1.0561		1.0576		1.0591		1.0554		1.0570		1.0585	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0655		1.0671		1.0687		1.0647		1.0663		1.0679		1.0639		1.0655		1.0671	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0748		1.0766		1.0783		1.0739		1.0756		1.0774		1.0729		1.0747		1.0764	
	1.04	0.06	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			1.0558		1.0573		1.0589		1.0552		1.0567		1.0582		1.0545		1.0560		1.0575	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0643		1.0659		1.0675		1.0636		1.0652		1.0668		1.0629		1.0644		1.0660	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0733		1.0750		1.0768		1.0724		1.0741		1.0758		1.0715		1.0732		1.0749	
1.5	1.02	0.06	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			1.0315		1.0330		1.0346		1.0308		1.0323		1.0338		1.0301		1.0316		1.0331	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0405		1.0421		1.0437		1.0397		1.0413		1.0429		1.0389		1.0405		1.0421	
		0.08	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0505		1.0522		1.0540		1.0495		1.0512		1.0530		1.0485		1.0502		1.0520	
	1.03	0.06	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4	12	4
			1.0304		1.0319		1.0335		1.0297		1.0313		1.0328		1.0291		1.0306		1.0321	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4

			1.0392	1.0408	1.0424	1.0384	1.0400	1.0416	1.0377	1.0392	1.0408
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0488	1.0505	1.0523	1.0478	1.0496	1.0513	1.0469	1.0486	1.0503
	1.04	0.06	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0294	1.0310	1.0325	1.0288	1.0303	1.0318	1.0281	1.0296	1.0311
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0380	1.0396	1.0412	1.0373	1.0388	1.0404	1.0365	1.0381	1.0397
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0472	1.0490	1.0507	1.0463	1.0480	1.0497	1.0454	1.0471	1.0488
1.55	1.02	0.06	12 4	13 4	13 4	12 4	12 4	13 4	12 4	12 4	12 4
			1.0070	1.0085	1.0099	1.0062	1.0078	1.0093	1.0055	1.0071	1.0086
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0161	1.0177	1.0193	1.0152	1.0168	1.0184	1.0144	1.0160	1.0176
		0.08	13 5	14 5	14 4	13 4	13 4	14 4	13 4	13 4	13 4
			1.0262	1.0280	1.0297	1.0252	1.0270	1.0287	1.0243	1.0260	1.0277
	1.03	0.06	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0059	1.0074	1.0089	1.0052	1.0067	1.0082	1.0045	1.0060	1.0075
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0147	1.0163	1.0179	1.0139	1.0155	1.0171	1.0132	1.0147	1.0163
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0245	1.0263	1.0280	1.0236	1.0253	1.0270	1.0226	1.0243	1.0260
	1.04	0.06	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0049	1.0064	1.0078	1.0042	1.0057	1.0071	1.0035	1.0050	1.0065
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0135	1.0151	1.0167	1.0127	1.0143	1.0159	1.0120	1.0135	1.0151
		0.08	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0229	1.0246	1.0264	1.0220	1.0237	1.0254	1.0211	1.0227	1.0244

Table 3: Impact of Parameter on the Optimal Values for fixed λ=1

λ = 1	γ		0.308			0.31			0.312		
θ			0.0055	0.006	0.0065	0.0055	0.006	0.0065	0.0055	0.006	0.0065
μ	α	β									
1.45	1.02	0.06	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.1101	1.1117	1.1134	1.1093	1.1110	1.1126	1.1086	1.1102	1.118
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.1209	1.1227	1.1245	1.1200	1.1218	1.1235	1.1190	1.1208	1.1226
		0.08	14 5	14 5	14 5	14 5	14 5	14 5	14 5	14 5	14 5
			1.1305	1.1324	1.1344	1.1295	1.1315	1.1334	1.1286	1.1305	1.1325
	1.03	0.06	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.1088	1.1105	1.1121	1.1081	1.1097	1.1113	1.1074	1.1090	1.1106
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.1193	1.1210	1.1228	1.1183	1.1201	1.1219	1.1174	1.1192	1.1209
		0.08	14 5	14 5	14 5	14 5	14 5	14 5	14 5	14 5	14 5
			1.1289	1.1309	1.1328	1.1280	1.1299	1.1319	1.1271	1.1290	1.1309
	1.04	0.06	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.1077	1.1093	1.1109	1.1070	1.1086	1.1102	1.1063	1.1079	1.1095
		0.07	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.1177	1.1194	1.1212	1.1168	1.1185	1.1203	1.1159	1.1176	1.1194
		0.08	14 5	14 5	14 5	14 5	14 5	14 5	14 5	14 5	14 5
			1.1275	1.1294	1.1314	1.1266	1.1285	1.1304	1.1258	1.1277	1.1295
1.5	1.02	0.06	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0824	1.0840	1.0856	1.0816	1.0832	1.0848	1.0808	1.0825	1.0841
		0.07	13 5	14 4	14 4	13 4	13 4	14 4	13 4	13 4	13 4
			1.0934	1.0952	1.0969	1.0925	1.0943	1.0960	1.0916	1.0933	1.0951

		0.08	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5
			1.1030		1.1049		1.1069		1.1020		1.1039		1.1059		1.1010		1.1030		1.1049	
	1.03	0.06	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0811		1.0827		1.0843		1.0803		1.0819		1.0835		1.0796		1.0812		1.0828	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0918		1.0935		1.0953		1.0908		1.0926		1.0943		1.0899		1.0916		1.0934	
		0.08	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5
			1.1014		1.1033		1.1052		1.1004		1.1024		1.1043		1.0995		1.1014		1.1033	
	1.04	0.06	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0799		1.0815		1.0831		1.0792		1.0808		1.0823		1.0785		1.0800		1.0816	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0901		1.0919		1.0936		1.0892		1.0910		1.0927		1.0883		1.0900		1.0918	
		0.08	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5
			1.0999		1.1018		1.1037		1.0990		1.1009		1.1028		1.0981		1.1000		1.1019	
1.55	1.02	0.06	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0566		1.0582		1.0598		1.0558		1.0574		1.0590		1.0550		1.0566		1.0582	
		0.07	14	5	14	5	14	5	14	5	14	5	14	5	14	4	14	4	14	4
			1.0676		1.0693		1.0711		1.0668		1.0685		1.0703		1.0660		1.0677		1.0694	
		0.08	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5
			1.0774		1.0793		1.0813		1.0764		1.0783		1.0802		1.0754		1.0773		1.0792	
	1.03	0.06	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0552		1.0568		1.0584		1.0545		1.0561		10576		1.0537		1.0553		1.0569	
		0.07	13	4	14	4	14	4	13	4	13	4	14	4	13	4	13	4	13	4
			1.0662		1.0679		1.0696		1.0652		1.0669		1.0687		1.0642		1.0660		1.0677	
		0.08	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5
			1.0757		1.0776		1.0796		1.0748		1.0767		1.0786		1.0738		1.0757		1.0776	
	1.04	0.06	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0540		1.0556		1.0571		1.0533		1.0548		1.0564		1.0525		1.0541		1.0556	
		0.07	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0645		1.0662		1.0679		1.0635		1.0653		1.0670		1.0626		1.0643		1.0660	
		0.08	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5	14	5
			1.0742		1.0761		1.0780		1.0733		1.0751		1.0770		1.0724		1.0742		1.0761	

Table 4: Effects of costs on the Optimal Values for fixed $c_h = 0.018$

$c_h = 0.018$	c_r		0.14						0.15						0.16					
c_{pr}			0.8		0.82		0.84		0.8		0.82		0.84		0.8		0.82		0.84	
c_s	c_o	c_n																		
1.45	1.01	0.015	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9912		0.9915		0.9919		0.9916		0.9919		0.9922		0.9919		0.9922		0.9925	
		0.017	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9915		0.9918		0.9921		0.9918		0.9921		0.9924		0.9922		0.9925		0.9928	
		0.019	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			0.9917		0.9920		0.9923		0.9921		0.9924		0.9927		0.9924		0.9927		0.9930	
	1.05	0.015	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0231		1.0234		1.0237		1.0235		1.0238		1.0241		1.0238		1.0241		1.0244	
		0.017	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.0234		1.0237		1.0240		1.0237		1.0240		1.0243		1.0240		1.0243		1.0247	
		0.019	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4	13	4
			1.023		1.023		1.024		1.023		1.024		1.024		1.024		1.024		1.024	

			6	9	2	9	2	6	3	6	9
	1.09	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0550	1.0553	1.0556	1.0553	1.0557	1.0560	1.0557	1.0560	1.0563
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0552	1.0556	1.0559	1.0556	1.0559	1.0562	1.0559	1.0562	1.0565
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0555	1.0558	1.0561	1.0558	1.0561	1.0564	1.0562	1.0565	1.0568
1.5	1.01	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9924	0.9927	0.9930	0.9927	0.9930	0.9934	0.9931	0.9934	0.9937
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9926	0.9930	0.9933	0.9930	0.9933	0.9936	0.9933	0.9936	0.9939
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9929	0.9932	0.9935	0.9932	0.9935	0.9938	0.9936	0.9939	0.9942
	1.05	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0243	1.0246	1.0249	1.0246	1.0249	1.0252	1.0250	1.0253	1.0256
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0245	1.0248	1.0251	1.0249	1.0252	1.0255	1.0252	1.0255	1.0258
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0248	1.0251	1.0254	1.0251	1.0254	1.0257	1.0254	1.0257	1.0261
	1.09	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0562	1.0565	1.0568	1.0565	1.0568	1.0571	1.0568	1.0572	1.0575
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0564	1.0567	1.0570	1.0567	1.0571	1.0574	1.0571	1.0574	1.0577
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0566	1.0570	1.0573	1.0570	1.0573	1.0576	1.0573	1.0576	1.0579
1.55	1.01	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9936	0.9939	0.9942	0.9939	0.9942	0.9945	0.9942	0.9945	0.9949
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9938	0.9941	0.9944	0.9941	0.9944	0.9948	0.9945	0.9948	0.9951
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			0.9940	0.9944	0.9947	0.9944	0.994	0.9950	0.9947	0.9950	0.9953
	1.05	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0254	1.0258	1.0261	1.0258	1.0261	0.0264	1.0261	1.0264	1.0267
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0257	1.0260	1.0263	1.0260	1.0263	1.0266	1.0264	1.0267	1.0270
		0.01	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4

		9	1.0259	1.0262	1.0265	1.0263	1.0266	1.0269	1.0266	1.0269	1.0272
	1.09	0.015	14 4	14 4	14 4	14 4	14 4	14 4	14 4	14 4	14 4
			1.0573	1.0576	1.0579	1.0576	1.0579	1.0583	1.0580	1.0583	1.0586
		0.017	14 4	14 4	14 4	14 4	14 4	14 4	14 4	14 4	14 4
			1.0575	1.0578	1.0582	1.0579	1.0582	1.0585	1.0582	1.0585	1.0588
		0.019	14 4	14 4	14 4	14 4	14 4	14 4	14 4	14 4	14 4
			1.0578	1.0581	1.0584	1.0581	1.0584	1.0587	1.0584	1.0588	1.0591

Table 5: Effects of costs on the Optimal Values for fixed $c_h = 0.02$

$c_h = 0.02$	c_r		0.14			0.15			0.16		
c_{pr}			0.8	0.82	0.84	0.8	0.82	0.84	0.8	0.82	0.84
c_s	c_o	c_n									
1.45	1.01	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0061	1.0064	1.0067	1.0064	1.0067	1.0070	1.0068	1.0071	1.0074
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0063	1.0066	1.0069	1.0067	1.0070	1.0073	1.0070	1.0073	1.0076
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0066	1.0069	1.0072	1.0069	1.0072	1.0075	1.0072	1.0076	1.0079
	1.05	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0380	1.0383	1.0386	1.0383	1.0386	1.0389	1.0386	1.0390	1.0393
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0382	1.0385	1.0388	1.0386	1.0389	1.0392	1.0389	1.0392	1.0395
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0385	1.0388	1.0391	1.0388	1.0391	1.0394	1.0391	1.0394	1.0397
	1.09	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0699	1.0702	1.0705	1.0702	1.0705	1.0708	1.0705	1.0708	1.0711
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0701	1.0704	1.0707	1.0704	1.0707	1.0711	1.0708	1.0711	1.0714
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0703	1.0706	1.0710	1.0707	1.0710	1.0713	1.0710	1.0713	1.0716
1.5	1.01	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0073	1.0076	1.0079	1.0076	1.0079	1.0082	1.0079	1.0082	1.0085
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0075	1.0078	1.0081	1.0078	1.0081	1.0084	1.0082	1.0085	1.0088
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0077	1.0080	1.0083	1.0081	1.0084	1.0087	1.0084	1.0087	1.0090
	1.05	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.039	1.039	1.039	1.039	1.039	1.040	1.039	1.040	1.040

			1	4	8	5	8	1	8	1	4
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0394	1.0397	1.0400	1.0397	1.0400	1.0403	1.0400	1.0404	1.0407
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0396	1.0399	1.0402	1.0400	1.0403	1.0406	1.0403	1.0406	1.0409
	1.09	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0710	1.0713	1.0716	1.0714	1.0717	1.0720	1.0717	1.0720	1.0723
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0713	1.0716	1.0719	1.0716	1.0719	1.0722	1.0719	1.0722	1.0725
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0715	1.0718	1.0721	1.0718	1.0721	1.0725	1.0722	1.0725	1.0728
1.55	1.01	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0084	1.0087	1.0090	1.0088	1.0091	1.0094	1.0091	1.0094	1.0097
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0087	1.0090	1.0093	1.0090	1.0093	1.0096	1.0093	1.0096	1.0099
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0089	1.0092	1.0095	1.0092	1.0095	1.0098	1.0096	1.0099	1.0102
	1.05	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0403	1.0406	1.0409	1.0406	1.0409	1.0413	1.0410	1.0413	1.0416
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0405	1.0408	1.0412	1.0409	1.0412	1.0415	1.0412	1.0415	1.0418
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0408	1.0411	1.0414	1.0411	1.0414	1.0417	1.0415	1.0418	1.0421
	1.09	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0722	1.0725	1.0728	1.0725	1.0728	1.0731	1.0729	1.0732	1.0735
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0724	1.0727	1.0730	1.0728	1.0731	1.0734	1.0731	1.0734	1.0737
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0727	1.0730	1.0733	1.0730	1.0733	1.0736	1.0733	1.0736	1.0740

Table 6: Effects of costs on the Optimal Values for fixed $c_h = 0.022$

$c_h = 0.022$	c_r		0.14			0.15			0.16		
c_{pr}			0.8	0.82	0.84	0.8	0.82	0.84	0.8	0.82	0.84
c_s	c_o	c_n									
1.45	1.01	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0202	1.0205	1.0208	1.0206	1.0209	1.0212	1.0209	1.0212	1.0215
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0205	1.0208	1.0211	1.0208	1.0211	1.0214	1.0212	1.0214	1.0217

		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0207	1.0210	1.0213	1.0211	1.0214	1.0217	1.0214	1.0217	1.0220
	1.05	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0524	1.0527	1.0530	1.0527	1.0530	1.0533	1.0530	1.0533	1.0536
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0526	1.0529	1.0532	1.0529	1.0532	1.0535	1.0533	1.0536	1.0539
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0528	1.0531	1.0534	1.0532	1.0535	1.0538	1.0535	1.0538	1.0541
	1.09	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0845	1.0848	1.0851	1.0848	1.0851	1.0854	1.0851	1.0854	1.0857
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0847	1.0850	1.0853	1.0851	1.0853	1.0856	1.0854	1.0857	1.0860
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0850	1.0853	1.0855	1.0853	1.0856	1.0859	1.0856	1.0859	1.0862
1.5	1.01	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0216	1.0219	1.0221	1.0219	1.0222	1.0225	1.0222	1.0225	1.0228
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0218	1.0221	1.0224	1.0221	1.0224	1.0227	1.0225	1.0228	1.0231
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0220	1.0223	1.0226	1.0224	1.0227	1.0230	1.0227	1.0230	1.0233
	1.05	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0537	1.0540	1.0543	1.0540	1.0543	1.0546	1.0543	1.0546	1.0549
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0539	1.0542	1.0545	1.0542	1.0545	1.0548	1.0546	1.0549	1.0552
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0542	1.0544	1.0547	1.0545	1.0548	1.0551	1.0548	1.0551	1.0554
	1.09	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0858	1.0861	1.0864	1.0861	1.0864	1.0867	1.0865	1.0868	1.0870
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0860	1.0863	1.0866	1.0864	1.0867	1.0870	1.0867	1.0870	1.0873
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0863	1.0866	1.0869	1.0866	1.0869	1.0872	1.0869	1.0872	1.0875
1.55	1.01	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0229	1.0232	1.0235	1.0232	1.0235	1.0238	1.0235	1.0238	1.0241
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.023	1.023	1.023	1.023	1.023	1.024	1.023	1.024	1.024

			1	4	7	4	7	0	8	1	4
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0234	1.0236	1.0239	1.0237	1.0240	1.0243	1.0240	1.0243	1.0246
	1.05	0.015	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0550	1.0553	1.0556	1.0553	1.0556	1.0559	1.0557	1.0559	1.0562
		0.017	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0552	1.0555	1.0558	1.0556	1.0559	1.0561	1.0559	1.0562	1.0565
		0.019	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4	12 4
			1.0555	1.0558	1.0561	1.0558	1.0561	1.0564	1.0561	1.0564	1.0567
	1.09	0.015	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0870	1.0873	1.0877	1.0874	1.0877	1.0880	1.0877	1.0880	1.0883
		0.017	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0873	1.0876	1.0879	1.0876	1.0879	1.0882	1.0879	1.0883	1.0886
		0.019	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4	13 4
			1.0875	1.0878	1.0881	1.0879	1.0882	1.0885	1.0882	1.0885	1.0888

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